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# Astro Unparticle Physics

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## Abstract

We investigate the effects of all flavor blind CP-conserving unparticle operators on 5th force experiments, stellar cooling, supernova explosions and compare the limits with each other and with those obtainable from collider experiments. In general, astrophysical bounds are considerably stronger, however they depend strongly on the dimension  $d_{\mathcal{U}}$  of the unparticle operator. While for  $d_{\mathcal{U}} = 1$ , 5th force experiments yield exceedingly strong bounds, the bounds from stellar and supernova cooling are more comparable for  $d_{\mathcal{U}} = 2$ , with stellar cooling being most restrictive. Bounds on vectorial unparticle couplings are generally stronger than those on scalar ones.

## 1 Introduction

Recently the possible existence of a non-trivial scale-invariant sector with a non-trivial fixed point was proposed by Georgi [1]. These new fields, which couple weakly to Standard Model (SM) particles, are quite different from other extensions of the SM as they are not described in terms of particles but rather by "unparticles". A different, but in effect similar deviation from the standard model has been proposed by Van der Bij [2]. The picture is valid up to a certain scale, above which the picture changes. At low energies the unparticle sector is characterized by a scaling dimension  $d_{\mathcal{U}}$ , which is in general non-integer.

In this paper we want to assess possible effects of this extension of the standard model in astrophysics. There are by now several studies in this direction [3, 4]; in this paper we combine the different manifestations and give also a more detailed and complete treatment of the various unparticle operators. If the conformal invariance is not broken in the infrared, as it is assumed throughout this paper, astrophysical constraints can highly restrict the interactions between unparticle and SM fields.

We consider only couplings between SM singlet unparticles and Standard Model fields through CP-conserving and flavor blind interactions. In Ref. [5] a list of operators composed of SM fields with dimensions 4 or less has been given. For our purpose we only need the couplings to fermions and gauge bosons. With fermions we have:

$$\mathcal{L}_{Uff} = \frac{c_V}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} \gamma_{\mu} f O_{\mathcal{U}}^{\mu} + \frac{c_A}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f} \gamma_{\mu} \gamma_5 f O_{\mathcal{U}}^{\mu} + \frac{c_{S1}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{f} \not{D} f O_{\mathcal{U}} + \frac{c_{S2}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{f} \gamma_{\mu} f \partial^{\mu} O_{\mathcal{U}} \quad (1)$$

$$\begin{aligned} &+ \frac{c_{P1}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{f} \not{D} \gamma_5 f O_{\mathcal{U}} + \frac{c_{P2}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{f} \gamma_{\mu} \gamma_5 f \partial^{\mu} O_{\mathcal{U}} \\ \equiv & \frac{c_V}{M_Z^{d_{\mathcal{U}}-1}} \bar{f} \gamma_{\mu} f O_{\mathcal{U}}^{\mu} + \frac{c_A}{M_Z^{d_{\mathcal{U}}-1}} \bar{f} \gamma_{\mu} \gamma_5 f O_{\mathcal{U}}^{\mu} + \frac{c_{S1}}{M_Z^{d_{\mathcal{U}}}} \bar{f} \not{D} f O_{\mathcal{U}} + \frac{c_{S2}}{M_Z^{d_{\mathcal{U}}}} \bar{f} \gamma_{\mu} f \partial^{\mu} O_{\mathcal{U}} \quad (2) \\ &+ \frac{c_{P1}}{M_Z^{d_{\mathcal{U}}}} \bar{f} \not{D} \gamma_5 f O_{\mathcal{U}} + \frac{c_{P2}}{M_Z^{d_{\mathcal{U}}}} \bar{f} \gamma_{\mu} \gamma_5 f \partial^{\mu} O_{\mathcal{U}}. \end{aligned}$$

Here the coefficients in eq. (2) have been scaled to a common mass, chosen as the  $Z$ -boson mass  $M_Z$ , so that the only unknown quantities are the dimensionless coupling constants  $c_X$ . In the above equations,  $D$  is the covariant derivative, which introduces four-particle couplings involving the photon through  $D_{\mu} = \partial_{\mu} + ieQ A_{\mu} + \dots$ . The term with  $c_{S2}$  does not contribute to physical processes due to current conservation.

For photons we have

$$\mathcal{L}_{U\gamma\gamma} = -\frac{c_{\gamma\gamma}}{4\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} F_{\mu\nu} F^{\mu\nu} O_{\mathcal{U}} - \frac{c_{\tilde{\gamma}\tilde{\gamma}}}{4\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} O_{\mathcal{U}} \quad (3)$$

$$\equiv -\frac{c_{\gamma\gamma}}{4M_Z^{d_{\mathcal{U}}}} F_{\mu\nu} F^{\mu\nu} O_{\mathcal{U}} - \frac{c_{\tilde{\gamma}\tilde{\gamma}}}{4M_Z^{d_{\mathcal{U}}}} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} O_{\mathcal{U}} \quad (4)$$

The term with  $c_{\tilde{\gamma}\tilde{\gamma}}$  has the same structure as the effective coupling of axions. Possible couplings to gluons are not considered.

In the following sections, we will consider the various standard tests of new particles and forces and reach our conclusions. Since unparticle phase space integration is more involved than for usual particles, we have added an appendix with some useful results.

## 2 Constraints from 5th force experiments

New massless or light degrees of freedom can mediate new forces between SM particles that lead to an effective modification of the Newtonian law of gravity [6, 7]. The most stringent constraints come from composition-dependent experiments, which were originally pioneered by Eötvös, Pekár and Fekete [8]. They make use of the fact that a new (fifth) force would in general act differently on different bodies of equal mass, depending on their chemical composition [9]. Limits for such an interaction have been derived on different length scales ranging from sub-meter to astronomical scales of order AU.

For the experimental analyses, the fifth force is typically parametrized by the potential

$$V_5(r) = \pm \xi G m^2 ({}_1H^1) B_i B_j \frac{e^{-r/\lambda}}{r}, \quad (5)$$

where the coupling constant  $\xi$  has been normalized to the gravitational interaction between two hydrogen  ${}_1H^1$  atoms, with  $G$  Newton's constant of gravity.  $B_{i,j}$  are the baryon numbers of the two test objects.

The potential for interaction due to vector unparticle exchange can be derived from its propagator [10, 11]

$$\Delta_F^{\mu\nu} \equiv \int d^4x e^{ipx} \langle 0 | T O_{\mathcal{U}}^\mu(x) O_{\mathcal{U}}^\nu(0) \rangle = i \frac{A_{d_{\mathcal{U}}}}{2 \sin(\pi d_{\mathcal{U}})} \frac{-g^{\mu\nu} + p^\mu p^\nu / p^2}{(-p^2 - i\epsilon)^{2-d_{\mathcal{U}}}}, \quad (6)$$

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})}. \quad (7)$$

By taking the Fourier transform of this propagator in the low-energy limit one obtains

$$V_{\mathcal{U}} = C \alpha_{\mathcal{U}} \frac{B_i B_j}{r^{2d_{\mathcal{U}}-1}}, \quad (8)$$

$$\begin{aligned} \alpha_{\mathcal{U}} &= \frac{c_V^2}{4\pi} M_Z^{2-2d_{\mathcal{U}}} \frac{A_{d_{\mathcal{U}}}}{\pi} \Gamma(2d_{\mathcal{U}} - 2) \\ &= \frac{\pi^{1/2}}{(2\pi)^{2d_{\mathcal{U}}}} c_V^2 M_Z^{2-2d_{\mathcal{U}}} \frac{\Gamma(d_{\mathcal{U}} - 1/2)}{\Gamma(d_{\mathcal{U}})}, \end{aligned} \quad (9)$$

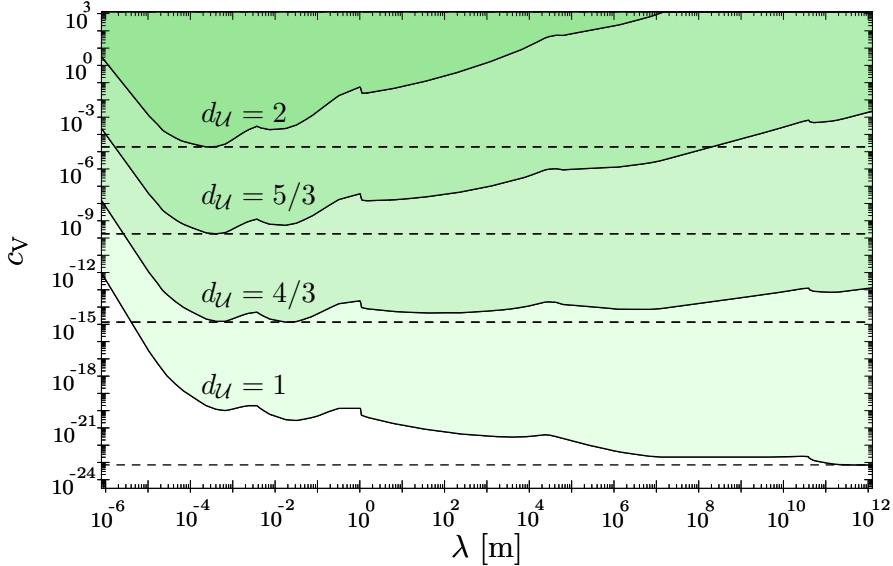
where  $C = \mathcal{O}(1)$  accounts for the quark and electron density inside the nucleons and atoms of the test objects. For a conservative limit, we take  $C \geq 1$ .

Also  $c_A$  contributes in the same way as  $c_V$ , up to a prefactor, and yields

$$\alpha_{\mathcal{U}} = \frac{(3\pi)^{1/2}}{(2\pi)^{2d_{\mathcal{U}}}} c_A^2 M_Z^{2-2d_{\mathcal{U}}} \frac{\Gamma(d_{\mathcal{U}} - 1/2)}{\Gamma(d_{\mathcal{U}})}. \quad (10)$$

The scalar and pseudo-scalar interactions  $\propto c_{S2}, c_{P1}, c_{P2}$  do not contribute to long-range non-relativistic forces. However, the contribution from  $c_{S1}$  gives

$$\alpha_{\mathcal{U}} = \frac{\pi^{1/2}}{(2\pi)^{2d_{\mathcal{U}}}} c_{S1}^2 \frac{m_i m_j}{M_Z^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} - 1/2)}{\Gamma(d_{\mathcal{U}})}, \quad (11)$$



**Figure 1:** Limits on vector unparticle interactions from Eötvös-type fifth-force experiments at different length scales  $\lambda$ , for various scaling dimensions  $d_{\mathcal{U}}$ . The dashed lines indicate the overall limit derived from the whole  $\lambda$  range.

where  $m_{i,j}$  are the masses of the electrons and nucleons between which the interaction is exchanged. The major contribution here comes from the nucleons with  $m_{i,j} \approx m({}_1H^1) \approx \frac{1}{90} M_Z$ .

The experimental limits on an interaction of type eq. (5) can be applied to the unparticle force eq. (8) by observing that the constraints on eq. (5) come mainly from measurements at a length scale  $r \approx \lambda$ . For  $r \gg \lambda$ ,  $V_5$  is exponentially suppressed, while for  $r \ll \lambda$  the experiments are less sensitive [7]. Therefore the exclusion limit at length scale  $\lambda$  is

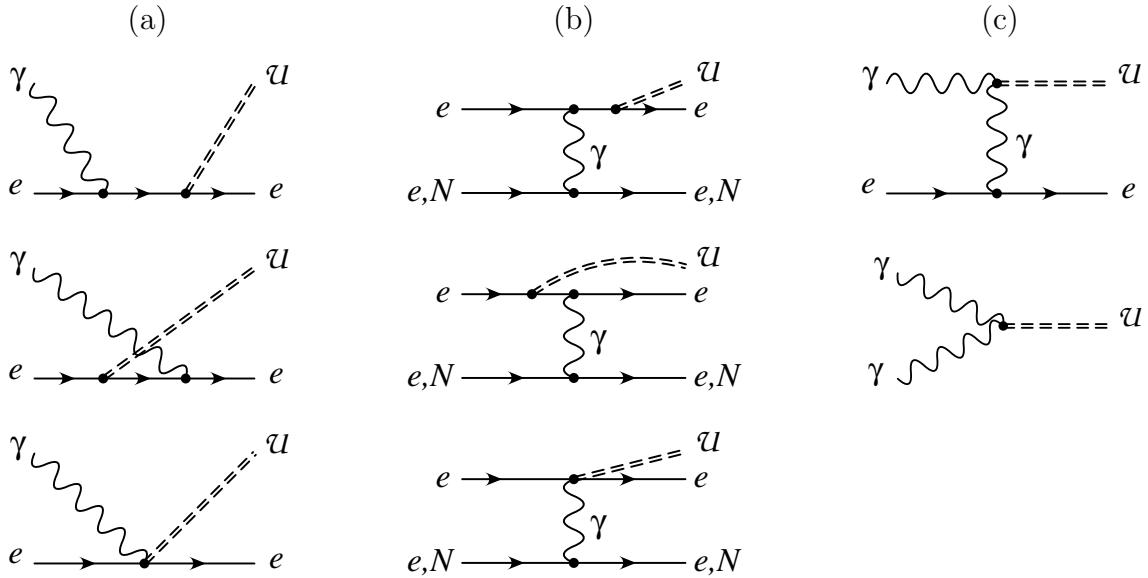
$$\alpha_{\mathcal{U},\text{lim}} \approx e^{-1} \xi_{\text{lim}} G m^2({}_1H^1) \lambda^{2d_{\mathcal{U}}-2}. \quad (12)$$

This result agrees well with the power-law analysis in Ref. [12] for  $d_{\mathcal{U}} = 2$ .

Taking the experimental values (see Ref. [7, 13] and references therein), results are shown for different scaling dimensions in Fig. 1. They can be readily translated to the axial-vector and scalar cases.

### 3 Constraints from stellar cooling

**Constraints from stellar cooling on fermion couplings.** In the hot and dense environment of stars, light weakly interacting particles can be produced efficiently and would contribute to the cooling of the star. Constraints on such particles can be derived from white dwarfs [14–16], the ignition condition for type I supernovae [17], horizontal-branch stars with a helium-burning core [18–20], and red giants near helium ignition flash [16, 21–23]. These processes have been studied extensively for axion emission, with the strongest bounds coming from helium-burning stars and red giants. In the following, we will focus on the evaluation



**Figure 2:** Feynman diagrams for unparticle emission in (a) Compton-like processes, (b) bremsstrahlung-like processes and (c) processes with unparticle-photon couplings.

of unparticle emission from helium-burning stars, which would lead to a reduction of the lifetime of the horizontal-branch stars.

Mainly two processes contribute to energy loss from horizontal-branch stars, the Compton process  $\gamma + e \rightarrow e + X$  and bremsstrahlung involving Hydrogen and Helium nuclei as well as electrons,  $e + H^+ \rightarrow e + H^+ + X$ ,  $e + He^{2+} \rightarrow e + He^{2+} + X$ ,  $e + e \rightarrow e + e + X$ . Here  $X$  is the axion or unparticle. The corresponding Feynman diagrams are shown in Fig. 2 (a) and (b).

The total cross-section for axion emission through the Compton process is

$$\sigma_a^c = \frac{\alpha g_{aee}^2}{3m_e^2} \left[ \frac{\omega}{m_e} \right]^2 \quad (13)$$

in the limit  $\omega \ll m_e$ , where  $\omega$  is the incoming photon energy.  $g_{aee}$  is the axion-electron coupling.

For unparticle emission, the calculations are somewhat more complicated. Due to the phase space factor  $A_{d_u} \theta(p_u^0) \theta(p_u^2) (p_u^2)^{d_u-2}$  [1], the final state integration requires some care. The important integrals are collected in the appendix. In the limit  $\omega \ll m_e$  one finds for the Compton process production of unparticles

$$\sigma_{u,V}^c = \frac{\alpha c_V^2}{m_e^2} \frac{2d_u}{(1+2d_u)\Gamma(2d_u)} \left[ \frac{\omega}{2\pi M_Z} \right]^{2d_u-2}, \quad (14)$$

$$\sigma_{u,A}^c = \frac{\alpha c_A^2}{m_e^2} \frac{2(2+d_u)}{(1+2d_u)\Gamma(2d_u)} \left[ \frac{\omega}{2\pi M_Z} \right]^{2d_u-2}, \quad (15)$$

$$\sigma_{u,S1}^c = \frac{\alpha c_{S1}^2}{M_Z^2} \frac{1}{(1+2d_u)\Gamma(2d_u)} \left[ \frac{\omega}{2\pi M_Z} \right]^{2d_u-2}, \quad (16)$$

$$\sigma_{\mathcal{U},\text{P1}}^c = \frac{4\pi^2\alpha c_{\text{P1}}^2}{m_e^2} \frac{(2+2d_{\mathcal{U}}+d_{\mathcal{U}}^2)}{(1+2d_{\mathcal{U}})(3+2d_{\mathcal{U}})\Gamma(2d_{\mathcal{U}})} \left[ \frac{\omega}{2\pi M_Z} \right]^{2d_{\mathcal{U}}}, \quad (17)$$

$$\sigma_{\mathcal{U},\text{P2}}^c = \frac{16\pi^2\alpha c_{\text{P2}}^2}{m_e^2} \frac{(2+2d_{\mathcal{U}}+d_{\mathcal{U}}^2)}{(1+2d_{\mathcal{U}})(3+2d_{\mathcal{U}})\Gamma(2d_{\mathcal{U}})} \left[ \frac{\omega}{2\pi M_Z} \right]^{2d_{\mathcal{U}}}, \quad (18)$$

In the hot environment of a star photons are generated thermically, with a distribution

$$n_{\gamma}(T, \omega) = \frac{\pi^{-2} \omega^2}{e^{\omega/T} - 1}. \quad (19)$$

The thermally averaged unparticle energy emission rate is then

$$Q(T)_{c,\mathcal{U}} = \int_0^\infty d\omega \omega n_e n_{\gamma} \sigma_{\mathcal{U}}^c(\omega), \quad (20)$$

with the electron density

$$n_e \approx \frac{1+X_{\text{H}}}{2} \frac{\rho}{m(\text{H})}, \quad (21)$$

where  $\rho$  is the total density and  $X_{\text{H}}$  the mass fraction of hydrogen. The averaging gives

$$\sigma_{\mathcal{U}}^c(\omega) = C \omega^r \quad \Rightarrow \quad Q_{\mathcal{U}}^c(T) = C n_e \frac{\zeta(4+r) \Gamma(4+r)}{\pi^2} T^{4+r}. \quad (22)$$

The emission rate for axion bremsstrahlung from electron-nucleus collisions is, in the limit for small incident electron velocities  $\beta_i \ll m_e$  [21, 24]

$$Q_{\text{a}}^{\text{eZ}}(\beta_i) = \frac{2}{135\pi m_e} Z^2 \alpha^2 g_{aee}^2 n_e n_z \beta_i^5, \quad (23)$$

where  $n_{e,z}$  are the electron and nucleus densities and  $Z$  is the proton number of the nucleus. For unparticle emission through bremsstrahlung in the non-relativistic limit one finds

$$Q_{\mathcal{U},\text{V}}^{\text{eZ}} = \frac{Z^2 \alpha^2 c_{\text{V}}^2 \beta_i}{m_e} n_e n_z \frac{-8(2+3d_{\mathcal{U}}) \csc(2\pi d_{\mathcal{U}})}{(2d_{\mathcal{U}}-1)(1+2d_{\mathcal{U}})(3+2d_{\mathcal{U}})\Gamma(2-2d_{\mathcal{U}})\Gamma(4d_{\mathcal{U}}-1)} \left[ \frac{m_e \beta_i^2}{\pi M_Z} \right]^{2d_{\mathcal{U}}-2}, \quad (24)$$

$$Q_{\mathcal{U},\text{A}}^{\text{eZ}} = \frac{Z^2 \alpha^2 c_{\text{A}}^2 \beta_i}{m_e} n_e n_z \frac{24(1-d_{\mathcal{U}}) \csc(2\pi d_{\mathcal{U}})}{(2d_{\mathcal{U}}-1)(1+2d_{\mathcal{U}})(3+2d_{\mathcal{U}})\Gamma(2-2d_{\mathcal{U}})\Gamma(4d_{\mathcal{U}}-1)} \left[ \frac{m_e \beta_i^2}{\pi M_Z} \right]^{2d_{\mathcal{U}}-2}, \quad (25)$$

$$Q_{\mathcal{U},\text{S1}}^{\text{eZ}} = \frac{Z^2 \alpha^2 c_{\text{S1}}^2 m_e \beta_i}{M_Z^2} n_e n_z \frac{5\pi^{-1/2}}{(2d_{\mathcal{U}}-1)^2(1+2d_{\mathcal{U}})(3+2d_{\mathcal{U}})\Gamma(2d_{\mathcal{U}}-1/2)} \left[ \frac{m_e \beta_i^2}{\pi M_Z} \right]^{2d_{\mathcal{U}}-2}, \quad (26)$$

$$Q_{\mathcal{U},\text{P1}}^{\text{eZ}} = \frac{Z^2 \alpha^2 c_{\text{P1}}^2 \beta_i}{m_e} n_e n_z \frac{-\pi^2(15+14d_{\mathcal{U}}+6d_{\mathcal{U}}^2) \csc(2\pi d_{\mathcal{U}})}{4(1+2d_{\mathcal{U}})(1+4d_{\mathcal{U}})\Gamma(-2d_{\mathcal{U}})\Gamma(4d_{\mathcal{U}})\Gamma(d_{\mathcal{U}}+7/2)} \left[ \frac{m_e \beta_i^2}{\pi M_Z} \right]^{2d_{\mathcal{U}}}, \quad (27)$$

$$Q_{\mathcal{U},\text{P2}}^{\text{eZ}} = \frac{Z^2 \alpha^2 c_{\text{P2}}^2 \beta_i}{m_e} n_e n_z \frac{-\pi^2(15+14d_{\mathcal{U}}+6d_{\mathcal{U}}^2) \csc(2\pi d_{\mathcal{U}})}{(1+2d_{\mathcal{U}})(1+4d_{\mathcal{U}})\Gamma(-2d_{\mathcal{U}})\Gamma(4d_{\mathcal{U}})\Gamma(d_{\mathcal{U}}+7/2)} \left[ \frac{m_e \beta_i^2}{\pi M_Z} \right]^{2d_{\mathcal{U}}}. \quad (28)$$

One can see that the rate for the axial vector vanishes for  $d_U \rightarrow 1$ ; below we will find this behaviour also for other processes. This results holds however only for the leading power in the velocity  $\beta_i$  in the non-relativistic limit, as one may see when expanding the correct expression in powers of  $\beta$ . The suppression can be understood from the fact that electron-nucleon scattering is independent of the chirality of the particles and therefore the  $L - R$  coupling is suppressed. Because however unparticles for  $d_U > 1$  carry a third polarization degree of freedom, the suppression is not total.

The bremsstrahlung emission rates have to be averaged over a Maxwellian distribution<sup>1</sup>

$$n_e(T, \beta_i) = \left(\frac{m}{2\pi T}\right)^{3/2} 4\pi \beta_i^2 \exp\left(-\frac{m\beta_i^2}{2T}\right) \quad (29)$$

so that

$$\begin{aligned} Q_U^{eZ}(\beta_i) &= C \beta_i^r & \Rightarrow & \quad Q_U^{eZ}(T) = \int_0^\infty d\beta_i n_e(T, \beta_i) Q_U^{eZ}(\beta_i) \\ & & & = C 2\pi^{-1/2} \Gamma\left(\frac{3+r}{2}\right) (2T/m_e)^{r/2}. \end{aligned} \quad (30)$$

Furthermore, summing over the relevant nuclei,

$$n_e \sum_z Z^2 n_z \approx n_e(n_{\text{H}} + 4n_{\text{He}}) \approx \frac{1 + X_{\text{H}}}{2} \left(\frac{\rho}{m_{\text{H}}}\right)^2. \quad (31)$$

Bremsstrahlung in electron-electron collisions leads to very similar results as bremsstrahlung in electron-nucleus collisions, except for the replacement  $Z^2 n_e n_z \rightarrow 4n_e^2$  in  $Q(\beta_i)$  or  $Z^2 n_e n_z \rightarrow \sqrt{2} n_e^2$  in  $Q(T)$ , respectively [21]. Bremsstrahlung in nucleus-nucleus collisions is negligible since the radiation of unparticles from nuclei with mass  $m_z$  is suppressed by powers of  $\beta_{i,z}/\beta_{i,e} \sim (m_e/m_z)^{1/2}$ .

The impact of weakly interacting particle emission on star cooling can be evaluated with a numerical code for stellar evolution [22, 23]. For simplicity, we give here the comparison of the unparticle emission rate to the axion emission constraints which have been analyzed earlier [18–20]. The relation between the two is summarized in Table 1.

One needs to observe that both bremsstrahlung and Compton processes play a role in red giant environments. At typical horizontal-branch star densities  $\rho \approx 0.6 \times 10^4 \text{ g/cm}^3$  and temperatures  $T \approx 10^8 \text{ K} = 8.6 \text{ keV}$ , the bremsstrahlung process contributes roughly 10% of the total axion emission rate, while the Compton process accounts for 90% of the rate [23]. Then the bound  $g_{aee} \lesssim 2 \times 10^{-13}$  [19, 20] translates into the limits in Table 2.

**Constraints from stellar cooling on photon couplings.** If unparticles only couple to photons, they would mainly contribute to star cooling through the process  $\gamma + e \rightarrow e + \mathcal{U}$  via t-channel photon exchange (usually called the Primakoff process), see Fig. 2 (c). In the

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<sup>1</sup>Since the density of horizontal-branch stars is relatively low, screening and degeneracy (Pauli blocking) effects are negligible [14, 23].

| $d_{\mathcal{U}}$   | 1   | 4/3   | 5/3   | 2  |
|---|---|---|---|--|
| $\frac{Q_{\mathcal{U},V}^c}{Q_a^c} \times 10^{12}$        | $3.34 \frac{c_V^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2}$    | $1.69 \frac{c_V^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-4/3}$    | $0.76 \frac{c_V^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2/3}$    | $0.32 \frac{c_V^2}{g_{aee}^2}$                                 |
| $\frac{Q_{\mathcal{U},A}^c}{Q_a^c} \times 10^{12}$        | $10.0 \frac{c_A^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2}$    | $4.22 \frac{c_A^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-4/3}$    | $1.67 \frac{c_A^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2/3}$    | $0.64 \frac{c_A^2}{g_{aee}^2}$                                 |
| $\frac{Q_{\mathcal{U},S1}^c}{Q_a^c} \times 10^{23}$       | $5.25 \frac{c_{S1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2}$ | $1.99 \frac{c_{S1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-4/3}$ | $0.72 \frac{c_{S1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2/3}$ | $0.25 \frac{c_{S1}^2}{g_{aee}^2}$                              |
| $\frac{Q_{\mathcal{U},P1}^c}{Q_a^c} \times 10^{12}$       | $31.4 \frac{c_{P1}^2}{g_{aee}^2}$                                 | $18.4 \frac{c_{P1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{2/3}$  | $9.66 \frac{c_{P1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{4/3}$  | $4.71 \frac{c_{P1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^2$ |
| $\frac{Q_{\mathcal{U},P2}^{eZ}}{Q_a^{eZ}} \times 10^{12}$ | $126 \frac{c_{P2}^2}{g_{aee}^2}$                                  | $73.5 \frac{c_{P2}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{2/3}$  | $38.6 \frac{c_{P2}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{4/3}$  | $18.8 \frac{c_{P2}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^2$ |
| $\frac{Q_{\mathcal{U},V}^{eZ}}{Q_a^{eZ}} \times 10^{12}$  | $118 \frac{c_V^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2}$     | $13.3 \frac{c_V^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-4/3}$    | $2.04 \frac{c_V^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2/3}$    | $0.36 \frac{c_V^2}{g_{aee}^2}$                                 |
| $\frac{Q_{\mathcal{U},A}^{eZ}}{Q_a^{eZ}} \times 10^{12}$  | 0   | $2.21 \frac{c_A^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-4/3}$    | $0.58 \frac{c_A^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2/3}$    | $0.14 \frac{c_A^2}{g_{aee}^2}$                                 |
| $\frac{Q_{\mathcal{U},S1}^{eZ}}{Q_a^{eZ}} \times 10^{23}$ | $185 \frac{c_{S1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2}$  | $17.4 \frac{c_{S1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-4/3}$ | $2.29 \frac{c_{S1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{-2/3}$ | $0.36 \frac{c_{S1}^2}{g_{aee}^2}$                              |
| $\frac{Q_{\mathcal{U},P1}^{eZ}}{Q_a^{eZ}} \times 10^{12}$ | $31.4 \frac{c_{P1}^2}{g_{aee}^2}$                                 | $9.22 \frac{c_{P1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{2/3}$  | $2.68 \frac{c_{P1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{4/3}$  | $0.77 \frac{c_{P1}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^2$ |
| $\frac{Q_{\mathcal{U},P2}^{eZ}}{Q_a^{eZ}} \times 10^{12}$ | $126 \frac{c_{P2}^2}{g_{aee}^2}$                                  | $36.9 \frac{c_{P2}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{2/3}$  | $10.7 \frac{c_{P2}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^{4/3}$  | $3.09 \frac{c_{P2}^2}{g_{aee}^2} \left(\frac{T}{M_Z}\right)^2$ |

**Table 1:** Comparison of unparticle emission rates to axion emission rates in a stellar plasma of temperature  $T$ . Separately shown are the rates from the Compton process ( $Q^c$ ) and the bremsstrahlung process ( $Q^{eZ}$ ), as well as different values of the scaling dimension  $d_{\mathcal{U}}$ .

limit  $\omega \ll m_e$  the cross-sections for this kind of process are

$$\sigma_{\gamma\gamma}^p = c_{\gamma\gamma}^2 / c_{\tilde{\gamma}\tilde{\gamma}}^2 \sigma_{\tilde{\gamma}\tilde{\gamma}}^p \quad (32)$$

$$= \frac{2\pi^2 \alpha c_{\gamma\gamma}^2}{\kappa^2} \left[ \frac{{}_3F_2(1, \frac{1+d_{\mathcal{U}}}{2}, 1 + \frac{d_{\mathcal{U}}}{2}; \frac{1}{2} + d_{\mathcal{U}}, 1 + d_{\mathcal{U}}; 4\omega^2/\kappa^2)}{\Gamma(2d_{\mathcal{U}})} \right. \\ \left. - \frac{d_{\mathcal{U}} {}_4F_3(1, \frac{1+d_{\mathcal{U}}}{2}, 1 + \frac{d_{\mathcal{U}}}{2}, 1 + d_{\mathcal{U}}; d_{\mathcal{U}}, \frac{1}{2} + d_{\mathcal{U}}, 2 + d_{\mathcal{U}}; 4\omega^2/\kappa^2)}{(1 + d_{\mathcal{U}})\Gamma(2d_{\mathcal{U}})} \right] \left[ \frac{\omega}{2\pi M_Z} \right]^{2d_{\mathcal{U}}}, \quad (33)$$

where  $\kappa$  is the inverse Debye-Hückel radius, which accounts for the screening of the Coulomb potential of the electron in a free stellar plasma [21, 25, 26].  ${}_pF_q$  are generalized hypergeometric functions.

In previous studies, limits have been derived for the coupling of axions to photons. By comparing the unparticle production cross-section to the cross-section for  $\gamma + e \rightarrow e + a$ ,

$$\sigma_a^p = \frac{\alpha g_{a\gamma\gamma}^2}{8} \left[ 1 - \left( 1 + \frac{\kappa^2}{4\omega^2} \right) \log \left( 1 + \frac{4\omega^2}{\kappa^2} \right) \right], \quad (34)$$

these limits can be translated to corresponding limits for the unparticle couplings. Using  $T \approx 10^8$  K = 8.6 keV,  $\kappa^2 = 7.5 \times 10^{-8}$  GeV $^2$  and  $g_{a\gamma\gamma} m_e < 5.5 \times 10^{-14}$  [21], we find the

limits in Table 3. Note that the dependence of the results on  $\kappa$  is very mild; changing  $\kappa^2$  by an order of magnitude changes the limits in Table 3 only by up to 20%. Therefore these results should be reliable even without a detailed numerical code for stellar evolution.

#### 4 Constraints from SN 1987A

Unparticle emission would also influence supernova cooling. This has been analyzed for vector unparticles in Ref. [3, 4]. Here the analysis in Ref. [4] is extended to derive limits for other types in unparticle couplings, as in eq. (2).

The observation of the length of the neutrino burst of the supernova SN 1987A puts a strong constraint on the allowed energy loss rate due to unknown very weakly interacting (un)particles [18],

$$Q_X \lesssim 3 \times 10^{33} \text{ erg cm}^{-3} \text{ s}^{-1}. \quad (35)$$

Several processes can contribute to unparticle emission from the supernova core. The dominant effect comes from neutron bremsstrahlung,  $n + n \rightarrow n + n + \mathcal{U}$ , while proton bremsstrahlung is less important since the proton density in supernova cores is smaller than the neutron density. In principle bremsstrahlung processes with electrons,  $e + n \rightarrow e + n + \mathcal{U}$  and  $e + e \rightarrow e + e + \mathcal{U}$ , can be important due to collinear enhancement. However, the collinear phase space region is suppressed due to strong Coulomb screening effects in the dense core plasma, see e.g. Ref. [21].

Since the supernova core temperature  $T \approx 30$  MeV is much smaller than the neutron mass, the neutron bremsstrahlung process factorizes into a "hard"  $nn$  collision process and "soft" unparticle radiation from one of the external neutrons. Here one can distinguish between the case when the bremsstrahlung coupling is insensitive to the nucleon spin (vector and scalar couplings of the unparticles to the quarks) [27] and when the bremsstrahlung emission couples to the nucleon spin (axial-vector and pseudo-scalar couplings) [28].

For vector and scalar unparticle-quark interactions, one finds in the non-relativistic limit

$$Q_{\mathcal{U},V}^{nn} = \frac{C c_V^2 m_n \beta_i^7}{32\pi^{3/2}} n_n^2 \sigma_0^{nn} \frac{39 + 1073d_{\mathcal{U}} + 228d_{\mathcal{U}}^2 + 60d_{\mathcal{U}}^3}{(2d_{\mathcal{U}} - 1)(1 + 2d_{\mathcal{U}})(3 + 2d_{\mathcal{U}})(5 + 2d_{\mathcal{U}})\Gamma(2d_{\mathcal{U}} + 5/2)} \left[ \frac{m_n \beta_i^2}{2\pi M_Z} \right]^{2d_{\mathcal{U}}-2}, \quad (36)$$

$$Q_{\mathcal{U},S1}^{nn} = \frac{C c_{S1}^2 m_n \beta_i^3}{\pi^{3/2}} n_n^2 \sigma_0^{nn} \frac{2(21 - 8d_{\mathcal{U}} + 55d_{\mathcal{U}}^2 + 31d_{\mathcal{U}}^3 + 6d_{\mathcal{U}}^4)}{(2d_{\mathcal{U}} - 1)(1 + 2d_{\mathcal{U}})(3 + 2d_{\mathcal{U}})(5 + 2d_{\mathcal{U}})\Gamma(2d_{\mathcal{U}} + 5/2)} \left[ \frac{m_n \beta_i^2}{2\pi M_Z} \right]^{2d_{\mathcal{U}}}, \quad (37)$$

where  $\beta_i$  is the incident neutron velocity and  $\sigma_0^{nn} \sim 25 \times 10^{-27} \text{ cm}^2$  is the typical  $nn$  scattering cross section at the given energy [27].  $n_n \approx 3 \times 10^{14} \text{ g cm}^{-3}$  denotes the neutron density.

Convolution with the Maxwellian thermal distribution gives

$$Q_{\mathcal{U},V}(T) = \frac{C c_V^2 T^{7/2}}{32\sqrt{2\pi^2} m_n^{5/2}} n_n^2 \sigma_0^{nn} \frac{(39 + 1073d_{\mathcal{U}} + 228d_{\mathcal{U}}^2 + 60d_{\mathcal{U}}^3)\Gamma(3 + 2d_{\mathcal{U}})}{(2d_{\mathcal{U}} - 1)(1 + 2d_{\mathcal{U}})(3 + 2d_{\mathcal{U}})(5 + 2d_{\mathcal{U}})\Gamma(2d_{\mathcal{U}} + 5/2)} \left[ \frac{T}{2\pi M_Z} \right]^{2d_{\mathcal{U}}-2}, \quad (38)$$

$$Q_{\mathcal{U},S1}^{nn}(T) = \frac{C c_{S1}^2 T^{3/2}}{\sqrt{2} \pi^2 m_n^{1/2}} n_n^2 \sigma_0^{nn} \frac{2(21 - 8d_{\mathcal{U}} + 55d_{\mathcal{U}}^2 + 31d_{\mathcal{U}}^3 + 6d_{\mathcal{U}}^4) \Gamma(3 + 2d_{\mathcal{U}})}{(2d_{\mathcal{U}} - 1)(1 + 2d_{\mathcal{U}})(3 + 2d_{\mathcal{U}})(5 + 2d_{\mathcal{U}}) \Gamma(2d_{\mathcal{U}} + 5/2)} \left[ \frac{T}{2\pi M_Z} \right]^{2d_{\mathcal{U}}}, \quad (39)$$

Our result for  $Q_{\mathcal{U},V}^{nn}(T)$  has the same dimensional dependence as in Ref. [4], but we are able to identify an additional numerical prefactor between 0.004 and 0.0014, depending on  $d_{\mathcal{U}}$ . Thus we arrive at somewhat weaker bounds for the unparticle interactions. In addition we obtain bounds for scalar interaction between unparticles and Standard Model fermions. Assuming  $C \geq 1$ , the bounds in Table 2 are obtained.

For the emission of axial-vector and pseduo-scalar unparticles, the matrix elements factorize in a similar way into the on-shell  $nn$  collision process and soft radiation from one of the external legs. Since the axial-vector and pseduo-scalar unparticle emission couples to the spins of the nucleons, one needs to take into account the spin dependence of the  $nn$  transition, which is given by the dynamical spin structure function [28, 29]. Following Ref. [28, 30], we obtain

$$Q_{\mathcal{U},A}^{nn}(T) = \frac{C c_A^2 T^2 n_n}{4\pi^2 \Gamma(2d_{\mathcal{U}})} \left[ \frac{T}{2\pi M_Z} \right]^{2d_{\mathcal{U}}-2} \int_0^\infty dx x^{2d_{\mathcal{U}}} e^{-x} \frac{\Gamma_\sigma/T}{x^2 + (\Gamma_\sigma/2T)^2}, \quad (40)$$

$$Q_{\mathcal{U},P1}^{nn}(T) = \frac{3 C c_{P1}^2 T^2 n_n}{4(1 + 2d_{\mathcal{U}}) \Gamma(2d_{\mathcal{U}})} \left[ \frac{T}{2\pi M_Z} \right]^{2d_{\mathcal{U}}} \int_0^\infty dx x^{2d_{\mathcal{U}}+2} e^{-x} \frac{\Gamma_\sigma/T}{x^2 + (\Gamma_\sigma/2T)^2}, \quad (41)$$

$$Q_{\mathcal{U},P2}^{nn}(T) = \frac{3 C c_{P2}^2 T^2 n_n}{(1 + 2d_{\mathcal{U}}) \Gamma(2d_{\mathcal{U}})} \left[ \frac{T}{2\pi M_Z} \right]^{2d_{\mathcal{U}}} \int_0^\infty dx x^{2d_{\mathcal{U}}+2} e^{-x} \frac{\Gamma_\sigma/T}{x^2 + (\Gamma_\sigma/2T)^2}, \quad (42)$$

where  $\Gamma_\sigma$  is the spin fluctuation rate. Using a one-pion exchange model for the nucleon scattering kernel, one obtains the estimte  $\Gamma_\sigma \approx 450$  MeV for the typical temperature and density inside the supernova core [29]. A more robust evaluation based on experimental nucleon scattering data [28] finds a smaller value for the spin structure function, which can be parametrized by using  $\Gamma_\sigma \approx 100$  MeV. Taking this value and  $C \geq 1$  as before, the bounds in Table 2 are derived.

## 5 Comparison to reach of collider experiments and conclusions

In Tables 2 and 3 we summarize our limits on unparticle couplings derived from astrophysical constraints. The bounds correspond to the 90% CL experimental error of the astrophysical observations<sup>2</sup>, for the case that only one of the unparticle couplings  $c_X$  is non-zero at a time. For comparison we also show earlier results for limits from current (LEP, Tevatron) and future colliders (LHC, ILC). To get an estimate of the possible reach of a future international linear collider (ILC), we have assumed that it can perform the same kind of measurements as LEP, but with a 1000 times higher luminosity. Of course, only a proper analysis can go beyond this order-of-magnitude assessment of the sensitivity of ILC. The blanks in the table

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<sup>2</sup>In the derivation of the stellar energy loss constraints, large systematic uncertainties could arise in the calculation of nuclear interactions and stellar evolution. Since these errors are difficult to quantify they have not been taken into account here.

indicate that no results are available from the literature for the given interaction. Some of the processes are not sensitive to a certain coupling, as denoted by a bar in the table.

It can be seen that the constraints for astrophysics are generally considerably stronger than those from colliders. The strongest bounds are for vector/axial couplings. For small  $d_{\mathcal{U}}$  limits on a 5th force are by far the dominant constraints; however for  $d_{\mathcal{U}}$  tending towards two all constraints become similarly important; here star cooling provides the strongest bound. For scalar and pseudoscalar couplings the bounds are generally weaker, which is mainly due to the higher dimensionality of the interaction operators. For  $d_{\mathcal{U}} = 1$ , the unparticle scaling behavior corresponds to a regular massless particle, so that our limits also apply to any model which includes a new massless scalar or vector particle (see also Ref. [31]).

For the unparticle-photon couplings, our bounds from star cooling are much stronger than the limits from supernova cooling, taken from Ref. [32]. These couplings could also be constrained by the process  $e^+e^- \rightarrow \gamma + \mathcal{U}$  at LEP and ILC, but this has not been analyzed so far.

This analysis is restricted to the leading CP-conserving and flavor-diagonal unparticle interactions. The astrophysical constraints are not sensitive to operators that involve flavor changing neutral currents, which can be tested in precision experiments at low energies, such as heavy-flavor mixing and decays [39–45], as well as to operators that only couple to third-generation fermions [46, 47], heavy gauge bosons [48] or the Higgs boson [37, 49, 50]. Furthermore, direct CP-violation in the unparticle operators [51] can lead to new effects, which cannot be tested in astrophysics.

| Coupling               | $c_V$               |                      |                      |                     | $c_A$                |                      |                      |                     |
|------------------------|---------------------|----------------------|----------------------|---------------------|----------------------|----------------------|----------------------|---------------------|
| $d_U$                  | 1                   | 4/3                  | 5/3                  | 2                   | 1                    | 4/3                  | 5/3                  | 2                   |
| 5th force ("Eötvös")   | $7 \cdot 10^{-24}$  | $1.4 \cdot 10^{-15}$ | $1.8 \cdot 10^{-10}$ | $2 \cdot 10^{-5}$   | $4 \cdot 10^{-24}$   | $8 \cdot 10^{-16}$   | $1 \cdot 10^{-10}$   | $1.1 \cdot 10^{-5}$ |
| Energy loss from stars | $5 \cdot 10^{-15}$  | $2.5 \cdot 10^{-12}$ | $1 \cdot 10^{-9}$    | $3.5 \cdot 10^{-7}$ | $6.3 \cdot 10^{-15}$ | $2 \cdot 10^{-12}$   | $7.3 \cdot 10^{-10}$ | $3 \cdot 10^{-7}$   |
| SN 1987A               | $1 \cdot 10^{-9}$   | $3.5 \cdot 10^{-8}$  | $1 \cdot 10^{-6}$    | $3 \cdot 10^{-5}$   | $2 \cdot 10^{-11}$   | $5.5 \cdot 10^{-10}$ | $1.5 \cdot 10^{-8}$  | $4.1 \cdot 10^{-7}$ |
| LEP                    | 0.005               | 0.045                | 0.04                 | 0.01                | 0.1                  | 0.045                | 0.04                 | 0.008               |
| Tevatron               |                     | 0.4                  | 0.05                 |                     |                      |                      |                      |                     |
| ILC                    | $1.6 \cdot 10^{-4}$ | $1.4 \cdot 10^{-3}$  | $1.3 \cdot 10^{-3}$  | $3.2 \cdot 10^{-4}$ | $3.2 \cdot 10^{-3}$  | $1.4 \cdot 10^{-3}$  | $1.3 \cdot 10^{-3}$  | $2.5 \cdot 10^{-4}$ |
| LHC                    |                     | 0.25                 | 0.02                 |                     |                      |                      |                      |                     |
| Electroweak precision  | 1                   | 0.2                  | 0.025                |                     | 1                    | 0.15                 | 0.01                 |                     |
| Quarkonia              |                     | 0.01                 | 0.1                  | 0.45                |                      |                      |                      |                     |
| Positronium            |                     | 0.25                 |                      |                     |                      | $2 \cdot 10^{-13}$   | $2 \cdot 10^{-8}$    | 0.03                |

II

| Coupling               | $c_{S1}$             |                      |                     |                     | $c_{P1}, 2c_{P2}$   |                     |                     |                   |
|------------------------|----------------------|----------------------|---------------------|---------------------|---------------------|---------------------|---------------------|-------------------|
| $d_U$                  | 1                    | 4/3                  | 5/3                 | 2                   | 1                   | 4/3                 | 5/3                 | 2                 |
| 5th force ("Eötvös")   | $6.5 \cdot 10^{-22}$ | $1.2 \cdot 10^{-13}$ | $1.6 \cdot 10^{-8}$ | $1.7 \cdot 10^{-3}$ | —                   | —                   | —                   | —                 |
| Energy loss from stars | $1.3 \cdot 10^{-9}$  | $7 \cdot 10^{-7}$    | $3 \cdot 10^{-4}$   | 0.13                | $4 \cdot 10^{-8}$   | $1.1 \cdot 10^{-5}$ | $3.3 \cdot 10^{-3}$ | 1                 |
| SN 1987A               | $8 \cdot 10^{-8}$    | $2.4 \cdot 10^{-6}$  | $6.6 \cdot 10^{-5}$ | $2 \cdot 10^{-3}$   | $5.5 \cdot 10^{-8}$ | $1.3 \cdot 10^{-6}$ | $3.5 \cdot 10^{-5}$ | $9 \cdot 10^{-4}$ |
| LEP                    | > 1                  | > 1                  | > 1                 | > 1                 | > 1                 | > 1                 | > 1                 | > 1               |
| ILC                    | > 1                  | > 1                  | > 1                 | > 1                 | > 1                 | > 1                 | > 1                 | > 1               |

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**Table 2:** Comparison of limits for unparticle-fermion couplings from astrophysical constraints and from present and future collider experiments. The astrophysical bounds have been derived in this work, while the collider bounds have been taken from the literature, as indicated by the references in the right column. Blank spaces are left where no results are available from the literature, while the bars denote that no bound on the coupling can be determined.

| Coupling               | $c_{\gamma\gamma}, c_{\tilde{\gamma}\tilde{\gamma}}$ |                      |                     |                     |
|------------------------|--|----------------------|---------------------|---------------------|
| $d_{\mathcal{U}}$      | 1  | 4/3                  | 5/3                 | 2                   |
| Energy loss from stars | $5.5 \cdot 10^{-14}$                                 | $1.7 \cdot 10^{-11}$ | $5.3 \cdot 10^{-9}$ | $1.7 \cdot 10^{-6}$ |
| SN 1987A               | $9 \cdot 10^{-7}$                                    | $4 \cdot 10^{-6}$    | $4 \cdot 10^{-5}$   | $8 \cdot 10^{-4}$   |

[32]

**Table 3:** Comparison of limits for unparticle-photon couplings from astrophysical constraints. The bounds from star cooling have been derived in this work, while the supernova bounds have been taken from the literature, as indicated by the reference in the right column.

### Note added

Shortly before finishing this manuscript, we became aware of related work on 5th force experiments [52] where similar, though weaker limits were obtained, since these authors included only results from Newtonian-law experiments at short but not at astronomical distances.

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### Appendix: Phase-space integrals

In the following the relevant phase-space integrals for the unparticle emission processes in this article are summarized.

For the Compton process  $e(p) + \gamma(k) \rightarrow e(p') + \mathcal{U}(k')$  in the non-relativistic limit, with the initial photon energy  $k_0 = \omega \ll m_e$  it is useful to choose a reference frame where the electron in the initial state is at rest. The phase space integration then yields

$$\begin{aligned} \sigma_{\mathcal{U}}^c &= \frac{A_{d_{\mathcal{U}}}}{4m_e\omega} \int \frac{d^3p'}{(2\pi)^3} \frac{1}{2m_e} \int \frac{d^4k'}{(2\pi)^4} \theta(k'_0)\theta(k'^2) (k'^2)^{d_{\mathcal{U}}-2} (2\pi)^4 \delta^{(4)}(k' + p' - k - p) |\mathcal{M}|^2 \\ &= \frac{A_{d_{\mathcal{U}}}}{32\pi^2 m_e^2 \omega} \int_0^1 d \cos \theta_{p'} \int_0^{2\omega \cos \theta_{p'}} dp' p'^2 (2\omega p' \cos \theta_{p'} - p'^2)^{d_{\mathcal{U}}-2} |\mathcal{M}|^2, \end{aligned} \quad (43)$$

where  $|\mathcal{M}|^2$  is the squared and spin-averaged matrix element, and  $\theta_{p'}$  is the angle between the incident photon and the outgoing electron. The following integrals appear:

$$\int_0^{2\omega \cos \theta_{p'}} dp' p'^{d_{\mathcal{U}}+n} (2\omega \cos \theta_{p'} - p')^{d_{\mathcal{U}}-2} = \frac{\Gamma(d_{\mathcal{U}}+1)\Gamma(d_{\mathcal{U}}+n+1)}{\Gamma(2d_{\mathcal{U}}+n)} (2\omega \cos \theta_{p'})^{2d_{\mathcal{U}}+n-1}, \quad (44)$$

$$\int_0^1 d \cos \theta_{p'} (\cos \theta_{p'})^{2d_{\mathcal{U}}+n} = \frac{1}{2d_{\mathcal{U}}+n+1} \quad (45)$$

with  $n = 1, 2, 3, \dots$

For bremsstrahlung  $e(p) + Z(q) \rightarrow e(p') + Z(q') + \mathcal{U}(k')$  one finds in the non-relativistic limit

$$\begin{aligned} \sigma_{\mathcal{U}}^{eZ} &= \frac{A_{d\mathcal{U}}}{4m_e m_z \beta_i} \int \frac{d^3 p'}{(2\pi)^3} \frac{1}{2m_e} \int \frac{d^3 q'}{(2\pi)^3} \frac{1}{2m_z} \int \frac{d^4 k'}{(2\pi)^4} \theta(k'_0) \theta(k'^2) (k'^2)^{d\mathcal{U}-2} \\ &\quad \times (2\pi)^4 \delta^{(4)}(k' + p' + q' - p - q) |\mathcal{M}|^2 \\ &= \frac{A_{d\mathcal{U}} m_e}{64\pi^4 m_z^2 \beta_i} \int_0^{\beta_i} d\beta_f \beta_f^2 \int \frac{d\Omega_{p'}}{4\pi} \int \frac{d\Omega_{k'}}{4\pi} \int_0^{m_e(\beta_i^2 - \beta_f^2)/2} d|\vec{k}'| |\vec{k}'|^2 \\ &\quad \times (\frac{1}{4} m_e^2 (\beta_i^2 - \beta_f^2)^2 - |\vec{k}'|^2)^{d\mathcal{U}-2} |\mathcal{M}|^2, \end{aligned} \quad (46)$$

where  $\beta_{i,f}$  are the velocity of the incoming and outgoing electron, respectively,  $\beta_i = |\vec{p}|/p_0$ ,  $\beta_f = |\vec{p}'|/p'_0$ , and  $m_z$  is the mass of the nucleus. After including the matrix element, the typical integrals are

$$\begin{aligned} \int_0^{\frac{m_e}{2}(\beta_i^2 - \beta_f^2)} d|\vec{k}'| |\vec{k}'|^n (\frac{1}{4} m_e^2 (\beta_i^2 - \beta_f^2)^2 - |\vec{k}'|^2)^{d\mathcal{U}-2} \\ = \frac{\Gamma(d\mathcal{U}-1)\Gamma(\frac{1+n}{2})}{2\Gamma(d\mathcal{U}+\frac{n-1}{2})} (\frac{m_e}{2}(\beta_i^2 - \beta_f^2))^{2d\mathcal{U}+n-4}, \end{aligned} \quad n = 1, 2, 3, \dots, \quad (47)$$

$$\int \frac{d\Omega_{k'}}{4\pi} [\vec{k}' \cdot (\vec{p} - \vec{p}')]^2 = \frac{1}{3} |\vec{k}'|^2 (\vec{p} - \vec{p}')^2, \quad (48)$$

$$\int \frac{d\Omega_{p'}}{4\pi} \frac{1}{(\vec{p} - \vec{p}')^2} = \frac{1}{m_e^2 \beta_i \beta_f} \log \frac{\beta_i + \beta_f}{\beta_i - \beta_f}, \quad (49)$$

$$\int \frac{d\Omega_{p'}}{4\pi} \frac{1}{(\vec{p} - \vec{p}')^4} = \frac{2}{m_e^4 (\beta_i^2 - \beta_f^2)^2}, \quad (50)$$

$$\int_0^{\beta_i} d\beta_f \beta_f^n (\beta_i^2 - \beta_f^2)^{2d\mathcal{U}} = -\frac{\pi \Gamma(\frac{1+n}{2}) \csc(2\pi d\mathcal{U})}{2\Gamma(-2d\mathcal{U})\Gamma(2d\mathcal{U}+\frac{n+3}{2})} \beta_i^{4d\mathcal{U}+n+1}, \quad n = 1, 2, 3, \dots, \quad (51)$$

$$\int_0^{\beta_i} d\beta_f \beta_f (\beta_i^2 - \beta_f^2)^{2d\mathcal{U}} \log \frac{\beta_i + \beta_f}{\beta_i - \beta_f} = \frac{\pi^{3/2} \csc(2\pi d\mathcal{U})}{2(2d\mathcal{U}+1)\Gamma(-2d\mathcal{U})\Gamma(2d\mathcal{U}+3/2)} \beta_i^{4d\mathcal{U}+2}. \quad (52)$$

Most of the above integrals are valid only for  $d\mathcal{U} \geq 1$ .

For bremsstrahlung off a neutron pair,  $n(p) + n(q) \rightarrow n(p') + n(q') + \mathcal{U}(k')$ , the situation is very similar to electron-nucleus-bremsstrahlung, albeit there are some differences due to fact that this process can only be evaluated by factorizing the strongly interacting  $nn \rightarrow nn$  scattering. In the center-of-mass frame

$$\begin{aligned} \sigma_{\mathcal{U}}^{nn} &= \frac{A_{d\mathcal{U}} \int d^3 p' \int d^3 q' \int d^4 k' \theta(k'_0) \theta(k'^2) (k'^2)^{d\mathcal{U}-2} \delta^{(4)}(k' + p' + q' - p - q) |\mathcal{M}|^2}{\int d^3 p' \int d^3 q' (2\pi)^4 \delta^{(4)}(p' + q' - p - q) |\mathcal{M}_0|^2} \\ &= \frac{A_{d\mathcal{U}} m_n}{4\pi^3 \beta_i} \int_0^{\beta_i} d\beta_f \beta_f^2 \int \frac{d\Omega_{p'}}{4\pi} \int \frac{d\Omega_{k'}}{4\pi} \int_0^{m_n(\beta_i^2 - \beta_f^2)} d|\vec{k}'| |\vec{k}'|^2 \\ &\quad \times (m_n^2 (\beta_i^2 - \beta_f^2)^2 - |\vec{k}'|^2)^{d\mathcal{U}-2} |\mathcal{M}|^2 / |\mathcal{M}_0|^2, \end{aligned} \quad (53)$$

where  $\mathcal{M}_0$  is the spin-averaged squared matrix element for  $nn$  scattering, which in the non-relativistic limit does not depend on the kinematic variables of the external particles, so that the integration in the denominator is trivial. As before,  $\beta_{i,f}$  are the velocities of the incoming and outgoing neutrons, respectively.

Besides the integrals eqs. (47) [with  $m_e/2 \rightarrow m_n$ ] and (51) one needs the integrals

$$\int \frac{d\Omega_{k'}}{4\pi} (\vec{k}' \cdot \vec{p})^{2n} = \frac{1}{2n+1} |\vec{k}'|^{2n} |\vec{p}'|^{2n}, \quad n = 1, 2, 3, \dots, \quad (54)$$

$$\int \frac{d\Omega_{k'}}{4\pi} (\vec{k}' \cdot \vec{p})^2 (\vec{k}' \cdot \vec{p}')^2 = \frac{1}{60} [(\vec{p} - \vec{p}')^4 + (\vec{p} + \vec{p}')^4] - \frac{1}{6} \int \frac{d\Omega_{k'}}{4\pi} [(\vec{k}' \cdot \vec{p})^4 + (\vec{k}' \cdot \vec{p}')^4]. \quad (55)$$

To arrive at the cross-section formulae in sections 3 and 4, relations between  $\Gamma$ -functions have been used extensively in some cases.

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